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Resonance of natural convection in a side heated enclosure with a mechanically oscillating bottom wall

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Abstract

An experimental study has been implemented to elucidate a resonance of natural convection in a side-heated enclosure with a mechanically oscillating bottom wall. The impetus of the present study is to provide an experimental verification of the resonant frequency of natural convection that has been numerically predicted so far. The experimental results show that the amplitude of fluctuating air temperature inside the enclosure peaks at a particular frequency of the bottom wall oscillation, which is indicative of resonance. The resonant frequency increases with the increase of the system Rayleigh number and it is little affected by the increase of forcing amplitude. The resonant frequency measured in the present experiment is in good accordance with the previous numerical predictions in which the models are based on the degree of thermal stratification in the interior.

1. Introduction

The transient behavior of natural convection in an enclosure has been extensively studied due to the relevance to many industrial applications. Earlier studies have focused on the heat-up behavior of internal fluid temperature in a side-heated enclosure. They clearly identified the initial oscillatory behavior of natural convection flow to the final steady state at moderate Rayleigh numbers [1–6].

Recently, much attention has been given to natural convection in an enclosure with a time-periodic external forcing [7–15]. Yang et al. [7] studied the effect of the sinusoidal time varying temperature oscillation of the hot and cold walls in natural convection and showed the variation of flow structure and the amplitude of heat transfer in a tall vertical enclosure. Kazmierczak and Chinoda [8] examined the effect of frequency and amplitude of oscillating hot wall temperature on the flow and heat transfer in an enclosure. Xia et al. [9] treated a similar problem and showed that the stability of flow field was strongly affected by the oscillation of hot wall temperature.

On the resonance of natural convection by an external thermal forcing, Lage and Bejan [10] and Anthohe and Bejan [11–13] performed theoretical and numerical works and reported the presence of resonance, which could be estimated in terms of the circulating period of flow in an enclosure. Such resonant phenomenon of natural convection was also comprehensively studied by Kwak and Hyun [14] and Kwak et al. [15]. Their studies clearly explained the resonance of natural convection by a wall thermal forcing. It was noted that the resonant frequency might be well predicted by the Brunt-Väisälä frequency, which was characterized by the thermal stratification of a system.

Further, other studies revealed that such resonance of natural convection could be also obtained by an external mechanical oscillation. Iwatsu et al. [16] employed an oscillatory top lid as a means of mechanical oscillation and showed a similar resonance of natural

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Nomenclature

A	dimensionless oscillating amplitude of bottom wall, $A = \Delta x/H$
D	depth of the enclosure (m)
f	oscillating frequency (Hz)
g	gravitational acceleration (m/s^2)
H	height of the enclosure (m)
k	thermal conductivity of air (W/m K)
L	width of the enclosure (m)
$\overline{Nu}$	space-averaged Nusselt number, $\overline{Nu} = q''H/k\Delta T$
p	oscillating period of bottom wall (s)
P	dimensionless period of oscillation, $P = p\alpha/H^2$
Pr	Prandtl number, $Pr = \nu/\alpha$
Ra	Rayleigh number, $Ra = g\beta\Delta TH^3/\alpha\nu$
Ra''	Rayleigh number in Fig. 2, $Ra'' = g\beta q''H^4/\alpha\nu k$
q''	heat flux (W/m^2)
$\overline{T}_h$	averaged hot wall temperature (K)
T_c	cold wall temperature (K)
t	time (s)
x, y, z	coordinates (m)
X, Y, Z	dimensionless coordinates, $X = x/H$, $Y = y/H$, $Z = z/H$

Greek symbols

α	thermal diffusivity (m^2/s)
β	thermal expansion coefficient (1/K)
Δx	oscillating displacement of the bottom wall (m)
ν	kinematic viscosity (m^2/s)
ΔT	temperature difference between hot and cold walls (K), $\Delta T = \overline{T}_h - T_c$
θ	dimensionless temperature, $\theta = (T - T_c)/(q''H/k)$
$\overline{\theta}$	dimensionless time-averaged temperature
θ'	fluctuating component of dimensionless temperature, $\theta = \overline{\theta} + \theta'$
τ	dimensionless diffusion time, $\tau = t\alpha/H^2$
ω	dimensionless frequency of oscillation, $\omega = fH^2/\alpha$

Subscripts

c	cold wall
h	hot wall
P	oscillating state value
S	steady state value

convection. Fu and Shieh [17,18] numerically investigated the effects of gravitational oscillation on the resonance and disclosed the heat transfer enhancement at a resonant frequency.

Up to date, however, most of the previously mentioned studies have been conducted theoretically and numerically. Only few experimental works could be found. Fobes et al. [19] implemented an experimental study to investigate heat transfer by a gravitational vibration of a water-filled column. Anthohe and Lage [11] performed an experiment on the natural convection in an enclosure with a time varying sidewall thermal boundary condition. However, clear experiment verification on the resonance of natural convection by external mechanical oscillation has not yet been fully accomplished.

Therefore, the present study aims to corroborate experimentally the presence of resonance of natural convection in a side-heated enclosure. To impose an external forcing into the enclosure, we construct a mechanical oscillating device that is attached to the bottom wall of the enclosure. Thus, the excitation inside the enclosure is purely mechanical by the external oscillation of the bottom wall. The impacts of the forcing frequency, the amplitude of the mechanical oscillation and the Rayleigh number on the resonance will be discussed in detail.

2. Experimental setup and test procedure

The experiments were conducted in a cubic enclosure of 300 mm in height, 300 mm in width and 300 mm in depth. The enclosure was made of four 10 mm thick Plexiglas plates and two 2 mm thick rubber plates as shown in Fig. 1. One side wall was electrically heated by a gold-coated film heater (Courtaulds performance Films, AU-ARE-12) to deliver a uniform heat flux. The opposed side wall consisted of a water jacket made of copper channels. Thus water from a cold water reservoir circulated this water jacket providing a constant cold temperature at the wall surface. The temperature variation along the cold wall surface was less than 1%.

In an effort to minimize heat loss through the walls, all the walls were deliberately insulated with 5 layers of polystyrene foam, 10 mm thick. At the back plate of the electrical heater, a 10 mm thick air gap was placed to reduce excessive heat loss. To estimate the conduction heat loss through the walls, we installed six pairs of K-type thermocouples (Omega Scientific, AWG 36) on the center of the inside and outside walls. By measuring the temperature difference from thermocouple pairs, therefore, we could evaluate the heat loss through the respective walls. Same technique was used to evaluate heat losses through the upper rubber wall and the moving

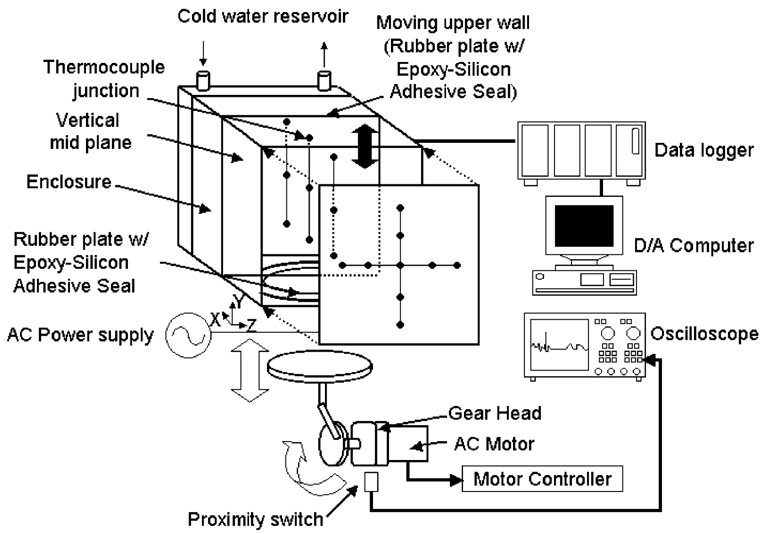

Fig. 1. A schematic view of experimental setup.

disk at the bottom wall. The total estimated heat loss was about 30.5% of the input heat flux. Therefore, only convective heat flux into the enclosure except for such heat loss was used to estimate the overall heat transfer rate.

Nine T-type thermocouples (Omega Scientific, AWG 40) were attached to the hot wall surface to measure the wall temperatures. In the core of the enclosure, nine T-type thermocouples of 25 μm diameter (Omega Scientific, SPCC-001, SPCP-001) were uniformly placed along the vertical cross section from the hot wall to cold wall to measure fluctuating air temperatures, as displayed in Fig. 1. The time constant of thermocouples was about 25 ms [20].

A crank-slide system driven by a DC motor and a speed reducing gear was installed at the bottom wall to produce low frequency oscillation. The top and bottom walls made of rubber plates were sealed by Epoxy-Silicon sealant to facilitate a free vertical movement of the bottom wall. Therefore, the top wall could move in tune with the bottom wall due to incompressibility of air.

Experiments were started by heating up the heater and by circulating cold water through the water jacket over 1 day without imposing oscillation. The steady state temperature data were collected by a multi-channel data acquisition system (Yokogawa, DA-100) and stored in a PC with the sampling frequency of 0.5 Hz. For the data acquisition of fluctuating air temperature, another high-speed data acquisition system (Realtime Device, TMX32) with the sampling rate of 80 Hz was used. After collecting the steady state temperature data, the oscillation of the bottom wall was imposed. During the bottom wall oscillation, the pulse signal produced by a proximity switch (Autonics, YS-2505-DNO) for each cycle was monitored by a digital oscilloscope (Lecroy, 9310A) to confirm the imposing frequency. After reaching a periodic steady state, the temperature data were stored by the data acquisition system for further analysis.

The uncertainties in the present experimental results were estimated by the single-sample experiment analysis by Kline and McClintock [21]. The estimated uncertainties for the non-dimensional temperatures were about 17.0% for $Ra = 7.3 \times 10^7$ and 14.6% for $Ra = 1.2 \times 10^8$ in 95% confidence level, respectively.

3. Results and discussion

In the present experiments, two cases of Rayleigh number, $Ra = 7.3 \times 10^7$ and $Ra = 1.2 \times 10^8$ were considered. The forcing frequency and amplitude were varied between $356 < \omega < 8556$ and $0.03 < A < 0.06$, respectively.

Fig. 2 depicts the vertical profiles of time-averaged internal air temperature for the steady non-oscillatory state and the oscillatory state at $(X, Z) = (0.5, 0.5)$. The steady state profiles obtained are qualitatively consistent with previous numerical results [22]. It is noted that the bottom wall oscillation makes the internal thermal stratification abate, compared to that for the steady non-oscillating state. Here, the forcing frequency ω and amplitude A were 713 and 0.06, respectively. This trend is more pronounced at the smaller Rayleigh number, $Ra = 7.3 \times 10^7$.

The temporal behavior of internal air temperature for various forcing frequency of the bottom-wall oscillation is displayed in Fig. 3. The Rayleigh number is

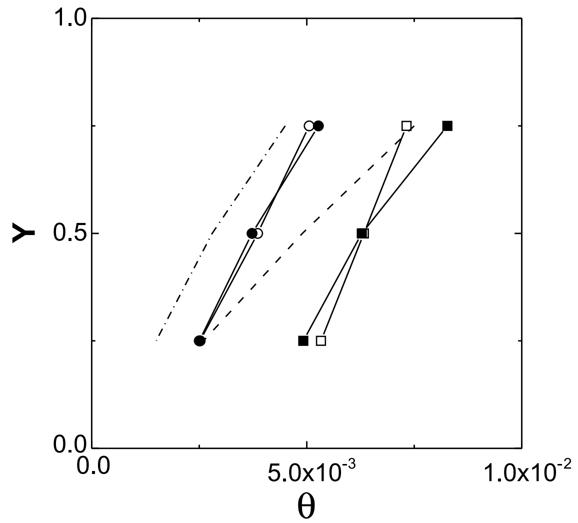

Fig. 2. Comparison of the vertical temperature profiles of internal air for the steady and the oscillating state ($\omega = 713$ and $A = 0.06$) at $(X, Z) = (0.5, 0.5)$. ■ and □ are for $Ra = 7.3 \times 10^7$, ● and ○ are for $Ra = 1.2 \times 10^8$; solid mark is for the steady state, hollow mark is for the oscillating state; the steady state numerical results of [21], ------ for $Ra'' = 7.0 \times 10^7$; – · – · – for $Ra'' = 10^8$.

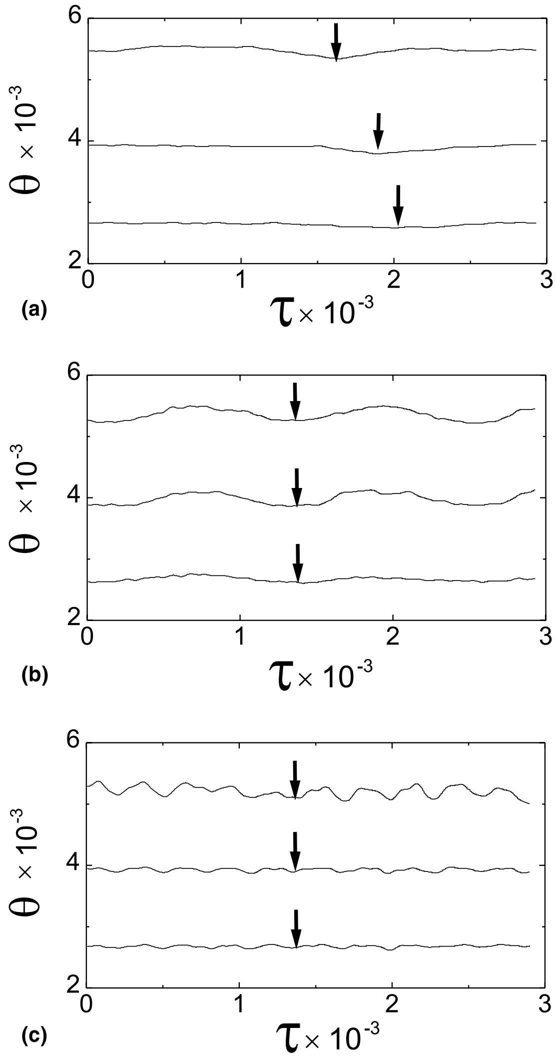

Fig. 3. Time-dependent behavior of non-dimensional temperature for various forcing frequency at $Ra = 1.2 \times 10^8$ and $A = 0.06$. The temperature are measured at $Y = 0.25$, 0.5, 0.75 from the bottom in the plane of $(X, Z) = (0.5, 0.5)$: (a) $\omega = 356$ and $P = 2.81 \times 10^{-3}$ ($p = 10$ s), (b) $\omega = 927$ and $P = 1.08 \times 10^{-3}$ ($p = 3.7$ s), (c) $\omega = 3566$ and $P = 2.8 \times 10^{-4}$ ($p = 1$ s).

fixed at $Ra = 1.2 \times 10^8$. Caused by the bottom-wall oscillation, the internal air temperature fluctuates despite of elevation in the enclosure. As the forcing frequency increases, the fluctuating amplitudes of temperature are substantially augmented at around $\omega = 927$ as portrayed in Fig. 3(b). It decreases with further increase of the forcing frequency. In addition, it is interesting to note that at a low forcing frequency, the air temperature at the upper portion inside the enclosure displays a phase-lag, compared to that at the lower portion as displayed in Fig. 3(a). However, the air temperatures are in phase at a specific frequency despite of the elevation as marked by arrows in Fig. 3(b). Such in-phase characteristic is maintained with further increase of the forcing frequency as shown in Fig. 3(c).

The temporal behavior of fluctuating component of air temperature at $(X, Y, Z) = (0.5, 0.5, 0.5)$ according to the forcing frequency is demonstrated in Fig. 4. The Rayleigh number and the forcing amplitude are fixed at $Ra = 1.2 \times 10^8$ and $A = 0.06$, respectively. The air temperatures fluctuate periodically by the sinusoidal oscillation of the bottom wall. As anticipated in Fig. 3, the fluctuation of air temperature is strongly intensified at $\omega = 927$. This behavior can be clearly observed in the power spectra of the temperature fluctuation (see Fig. 5). Further increase of the forcing frequency abates the fluctuating amplitude in Figs. 4(c) and 5(c).

By processing the Fast Fourier Transform of extensive experimental data, the variation of fluctuating amplitude according to the forcing frequency was obtained and displayed in Fig. 5. As the forcing frequency increases, the fluctuating amplitude shows a peak at $\omega = 927$ and decreases sharply beyond $\omega = 927$, as anticipated in Figs. 4 and 5. This trend is more substantial at the upper portion of the enclosure as shown in Fig. 6. The fluctuating amplitude at $Y = 0.75$ has other peaks at $\omega \sim 1368$ and $\omega \sim 3888$ which are considered as higher harmonics of the primary frequency of $\omega = 927$.

The variation of fluctuating amplitude at a lower Rayleigh number, $Ra = 7.3 \times 10^7$ is exhibited in Fig. 7.

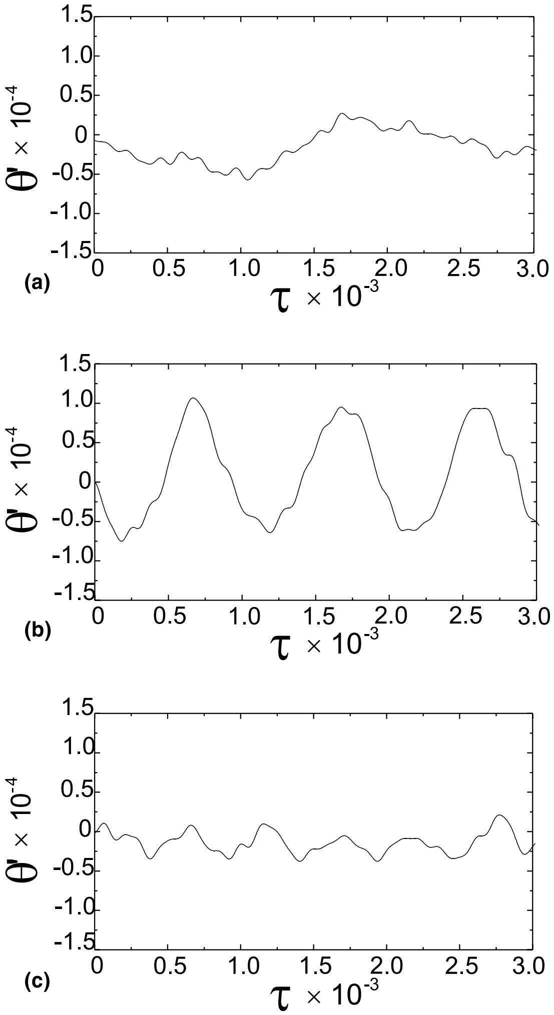

Fig. 4. Time-dependent behavior of the fluctuating component of temperature θ' for various forcing frequency at $(X, Y, Z) = (0.5, 0.5, 0.5)$ for $Ra = 1.2 \times 10^8$ and $A = 0.06$: (a) $\omega = 356$, (b) $\omega = 927$, (c) $\omega = 1997$.

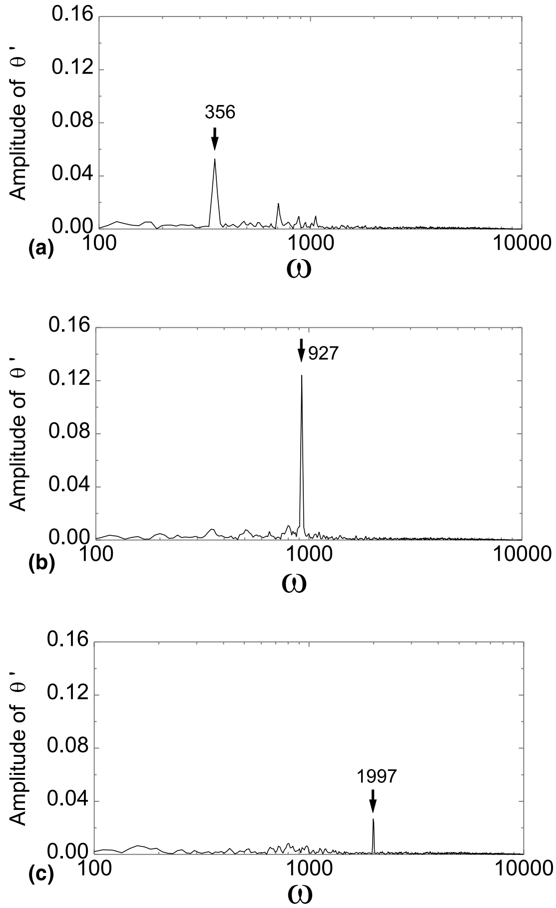

Fig. 5. Power spectra of air temperature fluctuation for various forcing frequency at $(X, Y, Z) = (0.5, 0.5, 0.5)$ for $Ra = 1.2 \times 10^8$ and $A = 0.03$: (a) $\omega = 356$, (b) $\omega = 927$, (c) $\omega = 1997$.

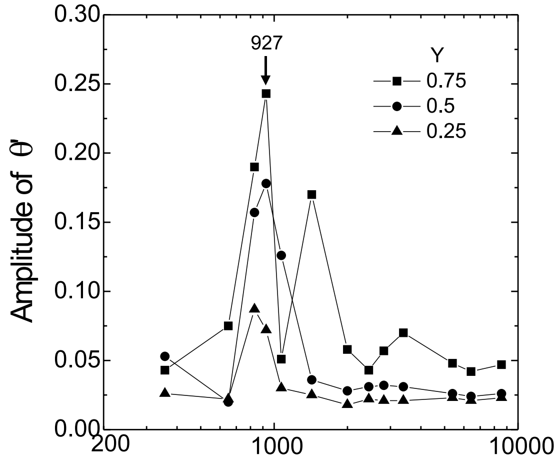

Fig. 6. Effect of the forcing frequency on air temperature fluctuation at $(X, Z) = (0.5, 0.5)$ for $Ra = 1.2 \times 10^8$ and $A = 0.03$.

The resonant frequency decreases to $\omega = 649$, while the general trend is similar to the case for the Rayleigh number of $Ra = 1.2 \times 10^8$ in Fig. 6. For the bottom-wall oscillation with the resonant frequency, the fluctuating amplitude shows a maximum at the mid-plane ($X = 0.5$) between the hot and cold walls. However, the resonant frequency was identical irrespective of the measuring locations. Therefore, it is now clear that the resonant frequency decreases with the decrease of the Rayleigh number.

The effect of oscillating amplitude of the bottom wall is shown in Fig. 8. Here, the Rayleigh number is $Ra = 1.2 \times 10^8$. As the oscillating amplitude A increases, the fluctuating amplitude monotonically increases.

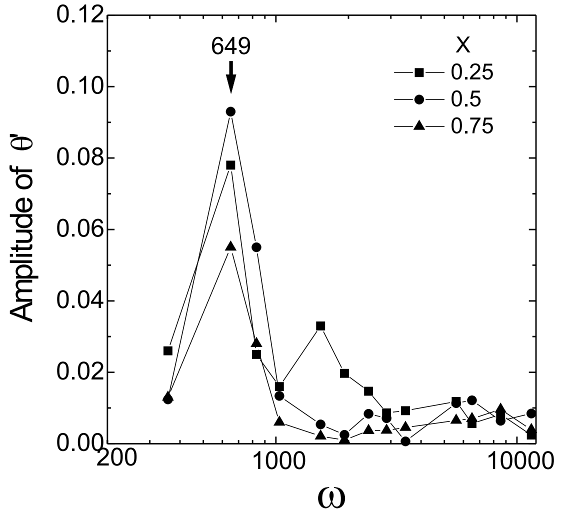

Fig. 7. Effect of the forcing frequency on air temperature fluctuation $(Y, Z) = (0.75, 0.5)$ for $Ra = 7.3 \times 10^7$ and $A = 0.03$.

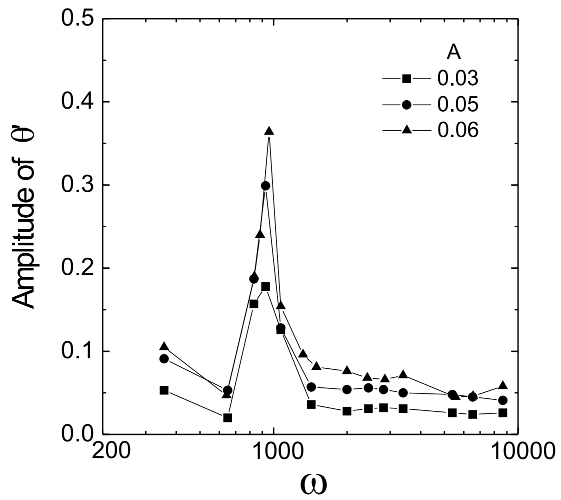

Fig. 8. Effect of the oscillation amplitude on air temperature fluctuation at $(X, Y, Z) = (0.5, 0.5, 0.5)$ for $Ra = 1.2 \times 10^8$.

However, the resonant frequency of $\omega = 927$ is not affected by the increase of the oscillating amplitude. This characteristic is in good accordance with the previous numerical findings [12,14,15].

The resonant frequencies obtained in the present experiment are compared with the previous numerical and theoretical predictions in Table 1. In the previous studies, the resonant frequencies were estimated in terms of the rotational period of a fluid wheel [10] and the internal gravity wave, i.e. Brunt-Väisälä frequency [4,14].

The rotational frequency in which a fluid parcel rotates along the vertical and horizontal walls in an enclosure can be expressed as

$$f = \frac{v}{2(L+H)}, \tag{1}$$

where L and H are length and height of the enclosure, respectively. The characteristic velocity v can be obtained by the scale analysis of Lage and Bejan [10]:

$$v \sim \frac{\alpha}{H}\left[\frac{L}{H}\left(\frac{RaPr}{\theta}\right)^{4/5}\right]^{1/2}. \tag{2}$$

Anthohe and Lage [12] modified the characteristic velocity by considering the horizontal wall:

$$v \sim \left[\frac{\alpha^2}{H^2}\frac{Ra}{\theta}\left(\frac{L}{H}\right)\left(\frac{RaPr}{\theta}\right)^{-1/5}\left(2+\frac{2}{1+L/H}\right)^{-1}\right]^{1/2}. \tag{3}$$

Paolucci and Chenoweth [4] estimated the resonant frequency based on the internal gravity wave in a square cavity:

$$f = \frac{0.95}{2\pi}\frac{\alpha}{H^2}\left(\frac{RaPr}{2}\right)^{1/2}. \tag{4}$$

Recently, Kwak et al. [22] predicted the resonant frequency based on the modified model of the internal gravity wave:

$$f = \frac{\sqrt{2}}{4\pi}\frac{\alpha}{H^2}\left(\frac{Ra}{\theta}Pr\frac{\partial\theta}{\partial Y}\right)^{1/2}. \tag{5}$$

As depicted in Table 1, the flow wheel models of Eqs. (1)–(3) result in slightly higher resonant frequencies. The resonant frequencies [Eqs. (4) and (5)] based on the internal gravity wave (i.e., the Brunt-Väisälä frequency)

Table 1
Comparison between the previous theoretical estimation and the present experimental result for the resonance frequency, ω

Ra	Theoretical prediction, Eqs. (1) and (2)	Theoretical prediction, Eqs. (1) and (3)	Theoretical prediction, Eq. (4)	Theoretical prediction, Eq. (5)	The present experimental result
7.3×10^7	1245	859	765	472	649
1.2×10^8	1932	1333	980	751	927

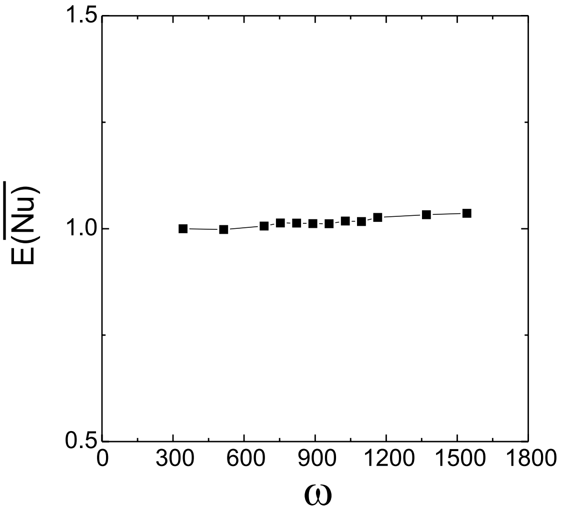

Fig. 9. The variation of the heat transfer enhancement factor for various forcing frequency ω at $Ra = 1.2 \times 10^8$ and $A = 0.06$.

are in fairly good agreement with the present experimental results. Therefore, it is concluded that the resonance occurs when the frequency of external forcing such as thermal and/or mechanical oscillation is in tune with the Brunt-Väisälä frequency, which is characterized by the degree of thermal stratification in a system.

It should be mentioned here that the resonant frequency predicted by the previous models [Eqs. (1)–(5)] is based on the two-dimensional computations. Some discrepancy could exist in comparison between the two-dimensional numerical results and the present three-dimensional experiment. However, a recent three-dimensional computation of Dol and Hanjalic [23] demonstrates a minor influence of the lateral walls in the mid-depth ($Z = 0.5$). Therefore, the resonant frequency measured in the present study might be compared with the previous numerical results with acceptable accuracy.

Fig. 9 portrays the effect of the forcing frequency on the time-averaged wall heat transfer. In order to assess the heat transfer enhancement by an external oscillation, the enhancement factor is defined as follows:

$$E(\overline{Nu}) = \frac{(\overline{Nu})_{\mathrm{P}}}{(\overline{Nu})_{\mathrm{S}}}. \tag{6}$$

In Fig. 9, the enhancement factor, E, is little affected by the bottom-wall oscillation in the limit of the frequency and amplitude range considered in the present study. This behavior is also in accordance with the previous numerical results for low amplitude oscillation [15]. The estimated uncertainty of the enhancement factor E was about 15.4% in 95% confidence level.

4. Conclusion

An experimental study has been conducted to verify the resonance of natural convection in a side-heated enclosure with a mechanically oscillating bottom wall. Impact of forcing frequency ($356 < \omega < 8556$) and amplitude ($0.03 < A < 0.06$) on the resonance was investigated.

The results obtained disclosed that the fluctuating amplitude of air temperature showed a peak at a specific forcing frequency, which was indicative of a resonance. As the Rayleigh number increased, the resonant frequency monotonically increased. However, it was little affected by the increase of the oscillating amplitude of the bottom wall. The resonant frequency measured in the present study was in fairly good agreement with the previous numerical predictions based on the natural frequency due to internal gravity wave. It was also noted that the time-averaged wall heat transfer rate was little changed according to the variation of forcing frequency.

Acknowledgements

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